ALGORITHMS OF NONSTATIONARY REGRESSION ESTIMATION IN SIGNALS PROCESSING

O.V. Krasotkina, V.V. Mottl, A.V. Kopylov[1].

A formal problem statement of nonstationary regression estimation in the case of constraints restricted the instant vector of regression coefficients is considered. This statement brings to the constrained quadratic optimization problem with pair-wise separable goal function. We produce two algorithms for optimization of such goal function: one based on quickest descent method and second based on dynamic programming algorithm. Both proposed algorithms have linear computational complexity in contrast to polynomial computational complexity of the quadratic programming problem of general kind.

Introduction

The variational approach [1] to constructing models of the nonstationary signals is a natural way of their parameterization in pattern recognition problems. In [1] it is considered a sufficiently wide class of problems of signals analysis along of axis of discrete argument $Y=(y_t, t=1,...,N)$, in which it is required for all point t to estimate the current values of sufficiently smoothly changing vector parameter $X=(\mathbf{x}_t, t=1,...,N)$ of certain local model. Such problems are particular case of nonstationary regression estimation problem. The problem of definition of nonstationary regression coefficients are set in [2] as quadratic optimization problem with pair-wise separable goal function

$$J(X)=\sum\nolimits_{t=1}^{N}\psi_t(\mathbf{x}_t)+\sum\nolimits_{t=2}^{N}\gamma_t(\mathbf{x}_{t-1},\mathbf{x}_t)\,, \tag{1}$$

which are structured as sums of partial functions, each of which depends only on one or two neighboring arguments. The natural way for the optimization of objective function of such a kind is the dynamic programming method [3]. In [2], the dynamic programming method is extended to the case of continues variables for quadratic objective functions by means introducing the concept of quadratic Bellman function. In addition, the concepts of the left and right Bellman functions are introduced and a new more efficiently dynamic programming procedure is offered. These procedure is called by authors "forward and against forward" procedure.

In applied problems there are often the natural constraints restricted the sought sequence of estimates. The goal of this paper is investigating realizability of dynamic programming procedure in the case of equality or non-equality constraints.

Non-stationary regression estimation problem under constraints

In this paper we consider a particular case of nonstationary regression problem with linear constraints when constraints are bring to sequence of separate constraints on each vector variable $\mathbf{x}_t\in\mathrm{R}^n$, $t=1,...,N$:

$$\mathbf{A}_t\mathbf{x}_t+\mathbf{c}_t\geq\mathbf{0}\,,. \tag{2}$$

$$\mathbf{B}_t\mathbf{x}_t+\mathbf{f}_t=\mathbf{0}\,. \tag{3}$$

Let us call the minimization problem of quadratic pair-wise separable objective function (1) with constraints (2)-(3) as pair-wise separable quadratic programming.

[1] Тульский государственный университет, 300600, г. Тула, пр-т. Ленина, 92, тел. 35-36-37, ф. 33-15-27, e-mail: mottl@uic.tula.ru, kopylov@uic.tula.ru ko180177@uic.tula.ru.

Let there are m non-equality and k, $k < n$ linearly independent equality. The presence only equality constraints do not complicate the problem. To overcome the presence of equality constraints it is enough to reduce the number of sought variables by k and treat as unknown only $n-k$ variable $\bar{\mathbf{x}}_t = (x_t^1,\ldots,x_t^{n-k})^T \in \mathrm{R}^{n-k}$, and others k are evaluated from (3). This denomination can be expressed in following form

$$\mathbf{x}_t = \mathbf{D}\bar{\mathbf{x}}_t + \mathbf{d}. \tag{4}$$

It can easily shown that such transformation leads to separable criterion absolutely equivalent to the criterion (1), and this criterion can be easily optimize by means dynamic programming procedure “forward and against forward”.

Let in addition to equality constraints there are non-equality constraints in our problem. Substitution the (4) in constraint (3) give equivalent system of inequalities for new variable $\bar{\mathbf{x}}_t$

$$\bar{\mathbf{A}}_t\bar{\mathbf{x}}_t + \bar{\mathbf{c}}_t \geq \mathbf{0}, \quad \bar{\mathbf{A}}_t = \mathbf{A}_t\mathbf{D}_t \left[n\times(n-k)\right], \quad \bar{\mathbf{c}}_t = \mathbf{A}_t\mathbf{d}_t + \mathbf{c}_t \in \mathrm{R}^n. \tag{5}$$

Therefore nonstationary regression estimation problem by both equality and non-equality constraints is lead to quadratic programming problem, consisted in minimization quadratic pair-wise separable objective function only with inequality constraints (5) restricted each variable individually. In this case minimization procedure in definition and main property Bellman function [2] both left and right is realized under constraints (5):

$$\begin{cases} \tilde{J}_t^-(\bar{\mathbf{x}}_t) = \min\limits_{\substack{g^i(\bar{\mathbf{x}}_{t-1})\geq 0\\ i=1,\ldots,n}} \left[\psi_t(\bar{\mathbf{x}}_t) + \gamma_t(\bar{\mathbf{x}}_{t-1},\bar{\mathbf{x}}_t) + \tilde{J}_{t-1}^-(\bar{\mathbf{x}}_{t-1})\right] = \psi_t(\bar{\mathbf{x}}_t) + F_t^-(\bar{\mathbf{x}}_t), \\ \tilde{J}_t^+(\mathbf{x}_t) = \min\limits_{\substack{g^i(\bar{\mathbf{x}}_N)\geq 0\\ i=1,\ldots,n}} \left[\psi_t(\bar{\mathbf{x}}_t) + \gamma_{t+1}(\bar{\mathbf{x}}_t,\bar{\mathbf{x}}_{t+1}) + \tilde{J}_{t+1}^+(\bar{\mathbf{x}}_{t+1})\right] = \psi_t(\bar{\mathbf{x}}_t) + F_t^+(\bar{\mathbf{x}}_t). \end{cases} \tag{6}$$

Here the functions $F_i^-(\mathbf{x}_i) = \min\limits_{g(\mathbf{x}_{i-1},\mathbf{x}_i)\geq 0} \left[\gamma_t(\mathbf{x}_{i-1},\mathbf{x}_i) + \tilde{J}_{i-1}^-(\mathbf{x}_{i-1})\right]$, $F_i^+(\mathbf{x}_i) = \min\limits_{g(\mathbf{x}_{i+1},\mathbf{x}_i)\geq 0} \left[\gamma_t(\mathbf{x}_{i+1},\mathbf{x}_i) + \tilde{J}_{i+1}^+(\mathbf{x}_{i+1})\right]$ are non-quadratic, because, first, the previous Bellman functions $\tilde{J}_{i-1}^-(\mathbf{x}_{i-1})$ and $\tilde{J}_{i+1}^+(\mathbf{x}_{i+1})$ are also non-quadratic and, second, they are formed by minimization under constraints. Therefore, in this case, the whole Bellman functions $\tilde{J}_i^-(\mathbf{x}_i)$ and $\tilde{J}_i^+(\mathbf{x}_i)$ will also be non-quadratic. This fact makes impossible numerical realization of the dynamic programming procedure, since the recurrent recalculation of parameters of the Bellman functions at each step becomes impossible.

The quickest descent method for pair-wise separable quadratic programming problem with inequality constraints

The first algorithm proposed in this problem is iterative and realized the quickest descent method for dual problem solution. The more complicated step of this algorithm is computing gradient of dual function on each iteration. Sought gradient is defined by minimization of Lagrangian function with Lagrangian coefficients obtained on previous step and without constraints. It can be easily shown, that in this case Lagrangian function is pair-wise separable and quadratic. Therefore its minimum point can be defined by means dynamic programming procedure “forward and against forward”, which computational complexity is proportional to the number of variable N. Therefore computational complexity of iterative algorithm of pair-wise separable quadratic programming is equivalent multifold using dynamic programming procedure with linear computational complexity and number of using is equal to number of iteration.

Approximate “forward and against forward” procedure of dynamic programming

Nevertheless, if we suppose that previous Bellman functions are quadratic, then the functions $F_i^-(\mathbf{x}_i)$ and $F_i^+(\mathbf{x}_i)$ can be obtained as result of minimization of respective quadratic functions under linear constraints (5). Both these functions and corresponding Bellman functions are not quadratic only because of the presence of non-equalities in constraints (5).

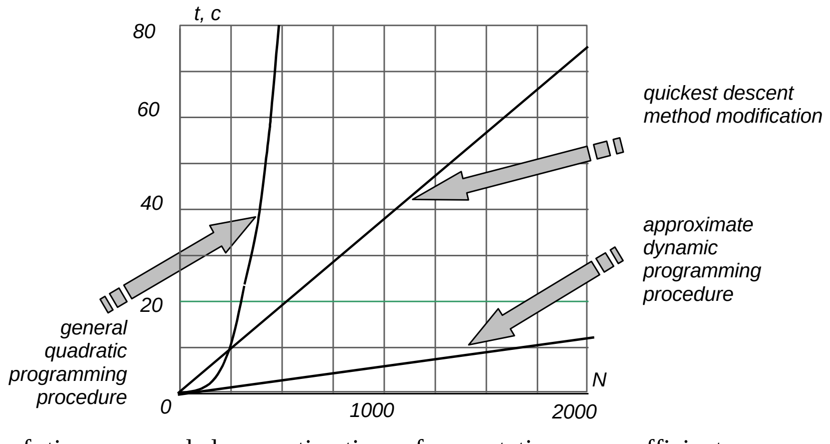
We consider here one way of overcoming this obstacle, namely, approximate realization of dynamic programming procedure, that consists in substituting functions $F_i^-(\mathbf{x}_i)$ and $F_i^+(\mathbf{x}_i)$ by appropriate quadratic functions

$$F_i'(\mathbf{x}_i) = b_i' + (\mathbf{x}_i - \mathbf{x}_i')^T \mathbf{Q}_i'(\mathbf{x}_i - \mathbf{x}_i') \cong F_i^-(\mathbf{x}_i),\ F_i''(\mathbf{x}_i) = b_i'' + (\mathbf{x}_i - \mathbf{x}_i'')^T \mathbf{Q}_i''(\mathbf{x}_i - \mathbf{x}_i'') \cong F_i^+(\mathbf{x}_i).$$

When such a substitution is done, the next Bellman functions will be quadratic (9), an approximate version of the dynamic programming procedure “forward against forward” will be possible.

Conclusion

In this paper two algorithms for the problem of pair-wise quadratic programming are proposed. Both algorithms have linear computational complexity in constraint to polynomial computational complexity of algorithms for quadratic programming problem of general kind. The first iterative algorithm is asymptotically exact and is based on quickest descent method with dynamic programming procedure on each step. Second non-iterative algorithm is approximate and based on single using of general dynamic programming procedure with replacing non-quadratic Bellman functions by their quadratic approximation. The dependence of time, expended on estimation of non-stationary coefficients sequence, from sequence length N.



The dependence of time, expended on estimation of non-stationary coefficients sequence, from sequence length N.

References

1. Mottl V.V., Blinov A.B., Kopylov A.V., Kostin A.A. Computer-aided signal and image processing: A universal variational approach. *Journal of Journals: Review of Global Scientific Achievements*, 1998, Vol. 2, No. 1, pp. 23-30.
2. Моттль В.В., Костин А.А., Красоткина О.В. Алгоритмы динамического программирования для анализа нестационарных сигналов. *Журнал вычислительной математики и математической физики*.
3. Bellman, R. *Dynamic Programming*. Princeton University Press, Princeton, N.J., 1957.